OUR THANKS TO THE STAFF AND CHILDREN AT THE CHILDREN'S AID SOCIETY IN NEW YORK, N.Y., FOR LETTING US SHOOT THE PHOTOS FOR THE COVER AND COVER STORY.

CREATIVE PATHWAYS TO MATH

by Douglas H. Clements, Ph.D.,
with Julie Sarama, Ph.D.
photos by James Levin

Children possess and build mathematical competencies from their first year and keep on learning mathematical ideas throughout their preschool years and beyond. This is not surprising. Mathematics helps children make more sense of their physical and social worlds. "That doesn't fit me—I grew too big!" "No fair! She has more than I do!"

Young children invent mathematical ideas and strategies. For example, take 5-year-old Alex:

Alex's brother, Paul, is 3. Alex bounds into the classroom and announces, "When Paul is 6, I'll be 8; when Paul is 9, I'll be 11; when Paul is 12, I'll be 14."

Teacher: My word! How on earth did you figure all that out?

Alex: It's easy. You just go "three-FOUR-five" [saying the "four" very loudly and clapping hands at the same time], you go "six-SEVEN [clap]-eight," you go "nine-TEN [clap!]-eleven"…..

This small but remarkable dialogue reflects the potential all young children have to learn—and even to create—mathematics.

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BEFORE THEY ENTER SCHOOL, MANY CHILDREN DEVELOP EARLY abilities in number and geometry, from accurate counting of objects to finding their way through their environment to making shapes. They use mathematical ideas in everyday life and develop informal mathematical knowledge that is surprisingly complex and sophisticated. With your guidance, children can become more acutely aware of this knowledge—an awareness that is crucial for mathematical understanding and learning.

Zachary's grandmother saw this awareness when she was walking him out of preschool. He stopped, pointed, and exclaimed, "Look, grandma! Hexagons! Hexagons all over the walk. You can put them together with no spaces!"

As such examples show, young children like doing mathematics. They all do. Boys and girls alike, in all socio-economic situations all exhibit spontaneous interest in mathematical ideas.

Young children can and should engage in mathematical thinking. All young children possess an informal knowledge of mathematics. Instruction should build upon and extend children's daily activities, interests, and questions, bringing the mathematics in such activities to the fore. This approach ensures that mathematical content will be meaningful for very young children.

You Hold the Key!

You can structure the classroom environment so that the potential for mathematics surrounds children. Show them the math in their everyday activities and plan special activities that focus on mathematics. Support their curiosity and offer appropriate challenges. You can:

- provide lots of unit blocks, along with time to use them.
- ask a child to get just enough scissors for every child who

Learning About Math—**PLAYFULLY!**

Play is often about mathematics. Nearly half of all the episodes of children's natural play observed by researchers included mathematics. This included:

- classification (putting away blocks in categories)
- magnitude ("This isn't big enough to cover the table.")
- enumeration (a boy says, "Look! I got 100!" and he and a friend count to check that estimate)
- dynamics (child stretches dough with her hand and makes a flat, circular shape)
- pattern and shape (a boy puts a double unit block on the rug, two unit blocks on the double block, and continues to build a symmetrical structure)
- spatial relations (telling a location or direction)

Children use math in everyday life and develop informed math knowledge.

is in the group.

■ challenge children to guess and check how many steps it is to the playground.

■ sit down with children in large and small groups to pose, solve, and discuss mathematical problems.

It's also important to make sure mathematically oriented materials such as blocks are readily available. Notice that moment when building mathematical language and concepts requires intervention. For example, when two children each claim that his building is the largest, you might discuss how one is "taller" but the other is "wider" (or "contains more blocks"). You may decide to add materials after observing children. For example, when you see children comparing the length of two rugs, make sure that connecting cubes, string, and other objects that might be used for measuring are close by.

Math Around the Room

You can help children connect their informal knowledge to their budding explicit knowledge of mathematics. For example, children might be able to manipulate blocks to find that adding one block to a group of three blocks results in a group of four blocks. Later, they can be asked to do similar problems even when the three blocks are hidden. Eventually, they will be able to "count on." Asked what two more than three is, they might say, "Threeeeee … four … five. Five!"

Children should also be encouraged to connect mathematics topics to each other. For example, children connect number to

geometry by counting the sides of shapes, using rows and columns to understand number combinations, or measuring the length of a rug. This helps strengthen concepts in these areas as well as beliefs about mathematics as a coherent system.

Our world can be better understood with mathematics. Early childhood is a good time for children to become interested in counting, sorting, building shapes, patterning, measuring, and estimating. *Quality preschool mathematics is not elementary arithmetic pushed down onto younger children.* Instead, it invites children to experience mathematics as they play in, describe, and think about their world.

Literacy Links to Math

Linking mathematics to literacy and other areas strengthens both. Most good mathematics activities also develop language and vocabulary. For example, when children are lining up, teachers can build in many opportunities to develop an understanding of mathematics. Children wearing something red can be asked to get in line first, those wearing blue to get in line second, and so on. Or, children wearing both something red and sneakers can be asked to head up the line.

Understanding stories involves mathematical understandings, such as conditionals (if/then), classification, patterning, order, and number. Think of the numbers, size relationships, sequences, and repetitious patterns in "Goldilocks and the Three Bears" and other favorite stories. It's no wonder that research shows that early mathematics experiences, especially geometric ones, result in later improvements in language and literacy, as well as general intelligence.

Connecting With Families

Here are some ways you can involve families in children's math learning:

■ Feature math nights. During these events, you might want to:

✔ Talk with families about your mathematics curriculum, including the wide range of mathematical concepts (see Chart: Development of Mathematical Concepts, page 42) and mathematical thinking that children will be involved in.

✔ Engage families in making some of the mathematics materials you'll be using (for example, cutting out colorful paper pattern blocks).

✔ Give families the opportunity to solve mathematical problems themselves, such as shape puzzle problems, so that they can experience the learning firsthand.

■ Have a "mathematics show" in which children share some of the mathematics projects they've been involved with. ECT

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10 CREATIVE WAYS TO TEACH MATH

Here are some activities for your classroom to add a bit of sparkle and creativity. As children work, ask critical questions such as "Did you try this?" "What would have happened if …?" "Do you think you could …?" to enhance children's understanding of mathematical ideas and vocabulary.

1 **Use dramatizations.** Invite children to pretend to be in a ball or box, feeling the faces, edges, and corners and to dramatize simple arithmetic problems such as: Three frogs jumped in the pond, then one more. How many are there in all?

2 **Use children's bodies.** Suggest that children show how many feet, mouths, and so on they have. Then invite children to show numbers with fingers, starting with the familiar, "How old are you?" to showing numbers in different ways.

3 **Use children's play.** Engage children in block play that allows them to do mathematics in numerous ways, including sorting, seriating, creating symmetric designs and buildings, making patterns, and so forth. Then introduce a game of Dinosaur Shop. Suggest that children pretend to buy and sell toy dinosaurs or other small objects.

4 **Use children's toys.** Encourage children to use "scenes" and toys to act out situations such as three cars on the road, or, later in the year, two monkeys in the trees and two on the ground.

5 **Use children's stories.** Share books with children that address mathematics but are also good stories (see Book Box, p.45). Later, help children see mathematics in any book.

6 **Use children's natural creativity.** Children's ideas about mathematics should be discussed with all children. Here's a "mathematical conversation" between two boys, each 6 years of age: "Think of the biggest number you can. Now add five. Then, imagine if you had that many cupcakes." "Wow, that's five more than the biggest number you could come up with!"

7 **Use children's problem-solving abilities.** Ask children to describe how they would figure out problems such as getting just enough scissors for their table or how many snacks they would need if a guest were joining the group.

8 **Use a variety of strategies.** Bring mathematics everywhere you go in your classroom, from counting children at morning meeting to setting the table, to asking children to clean up a given number of items.

9 **Use technology.** Try digital cameras to record children's mathematical work in their play and activities. Then use the photographs to aid class discussions, curriculum planning, and communication with parents. Use computers to mathematize situations and provide individualized instruction.

10 **Use assessments to measure children's mathematics learning.** Observations, discussions with children, and small-group activities help you learn about children's mathematical thinking and to make informed decisions about what each child can learn from future experiences.

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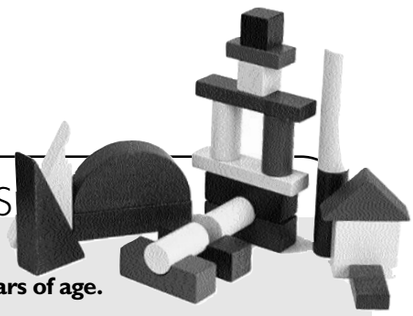

DEVELOPMENT OF MATHEMATICAL CONCEPTS

At about what age can children develop specific mathematics concepts? This chart outlines what children are capable of understanding at 3, 4, and 5 years of age.

NUMBER CONCEPT	AT 3 YEARS Children may:	AT 4 YEARS Children may:	AT 5 YEARS Children may:
Verbal Counting. Learning the standard sequences of number words	Count 1 to 10	Count one to 30, with emphasis on counting patterns; for instance, knowing that "21, 22 …" is parallel to "1, 2 …"	Count one to 100, with emphasis on patterns (e.g., "60, 70" is parallel to "6, 7;" "14" to "19" parallel to "4" to "9")
Object Counting. Creating a one-to-one correspondence between a number word and an item	Count one to four items, maintaining one-to-one correspondence	Count one to 10 items, knowing that the last counting word tells "how many"	Count one to 20 items
"Seeing" Numbers. Instantly "seeing how many" supports counting, comparing, and adding	See groups of one to three	See groups of one to five	See groups of one to six; regular patterns up to 10
Comparing Numbers. Comparing and ordering build on nonverbal knowledge and experience with collections	Identify whether collections are the "same" number or which is "more" visually	Use counting or matching to compare two collections one to five, despite appearances	Use counting to compare two collections one to 10, using words "equal," "more," "less," and "fewer"
Adding and Subtracting. Solving problems using informal strategies in math learning	Use nonverbal adding and subtracting with very small numbers of objects	Solve and make word problems using concrete modeling with sums to five	Pose and solve word problems using counting-based strategies such as counting on, sums to 10
GEOMETRY AND MEASUREMENT			
Shapes. Geometric shapes can be used to represent and understand objects	Match shapes, first with same size and orientation, then with different sizes and orientation	Recognize and name some variations of the circle, square, triangle, rectangle	Recognize and name circle, square, triangle, rectangle, in any size or orientation
Putting Together Shapes. Shapes can be decomposed and composed into other shapes and structures	Use shapes in isolation to make a picture	Cover an outline with shapes without leaving gaps by trial-and-error	Cover an outline with shapes without leaving gaps by using foresight. Make a picture by combining shapes
Locations, Directions, and Coordinates. Mathematics can precisely specify directions, routes, and locations	Understand and use ideas such as over, under, above, on, beside, next to, between	Learn a simple route from a map placed in direct relation to the space	Place toy objects in correct relative position to make a map of the classroom
Symmetry. Symmetry can be used to understand and create shapes in geometry and art	Show awareness of symmetry in block buildings	Informally create 2-D shapes and 3-D buildings that have symmetry	Identify and create shapes that have line or rotational symmetry
Measurement. Measuring can be used to specify and compare "how much"	Develop language such as bigger, longer, and taller	Discuss and compare attributes informally, including comparing gross differences	Compare length using another object. Measure with multiple copies of a unit (such as block)
Patterns. Patterns weave through all other topics in mathematics	Notice simple repeating patterns, such as a wall of blocks with long, short, long, short …	Copy simple repeating patterns	Notice and discuss patterns in arithmetic (such as adding one to any number results in the next "counting number")

MATERIALS AND **RESOURCES**

Here are some of the latest resources to help support your mathematics program:

BOOKS FOR TEACHERS

Engaging Young Children In Mathematics: Findings of the 2000 national conference on standards for preschool and kindergarten mathematics education by D.H. Clements, J. Sarama, & A.M. DiBiase, eds. In press. Mahwah, NJ: Lawrence Erlbaum Associates.

Mathematics in the Early Years by Juanita Copley (National Council of Teachers of Mathematics, 1999; $33.95)

The Young Child and Mathematics by Juanita Copley (NAEYC, 2000; $15)

ARTICLES FOR TEACHERS

Clements, D.H., & J. Sarama. 2000. Standards for preschoolers. *Teaching Children Mathematics* 7(1): 38–41.

Clements, D.H. 2001. Mathematics in the preschool. *Teaching Children Mathematics* 7: 270–75.

Clements, D.H., & J. Sarama. 2002. The role of technology in early childhood learning. *Teaching Children Mathematics* 8: 340–43.

Clements, D.H., J. Sarama, & A.M. DiBiase. 2002. Preschool and kindergarten mathematics: A national conference. *Teaching Children Mathematics* 8: 510–14.

Geist, E. 2001. Children are born mathematicians: Promoting the construction of early mathematical concepts in children under five. *Young Children* 56 (4): 12–19.

Murray, A. 2001. Ideas on manipulative math for young children. *Young Children* 56 (4): 28–29.

Sarama, J. (2002). Listening to teachers: Planning for professional development. *Teaching Children Mathematics,* 9, 36-39.

Teaching Children Mathematics: The Early Childhood Corner. The National Council of Teachers of Mathematics' (NCTM) journal dedicates this monthly "corner" to teachers of children before first grade.

WEBSITES

www.nctm.org National Council of Teachers of Mathematics

www.naeyc.org/resources/position_statements/psmath.htm Early Childhood Mathematics: Promoting Good Beginnings. A joint position statement of the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM).

standards.nctm.org The new math standards, Principles and Standards for School Mathematics, and many activities, Web-based software environments, and videos.

RESEARCH-BASED CURRICULA

Big Math for Little Kids™ (Ginsburg, Greenes, & Balfanz, 2003). Comprehensive mathematics curriculum for preschool children. *cgreenes@bu.edu*

Building Blocks—Foundations for Mathematical Thinking, PreKindergarten to Grade 2 The first products include a comprehensive preschool curriculum and integrated software (which can be obtained separately) (Clements & Sarama, 2003; Schiller et al., 2003); Pre-K to grade 2 products are being developed. *clements@buffalo.edu, jsarama@buffalo.edu* or *www.gse.buffalo.edu/org/buildingblocks*.

Creative Publications Math Curriculum Resources A group of supplementary books with stories that engage children in math problem solving. *caseyb@bc.edu.*

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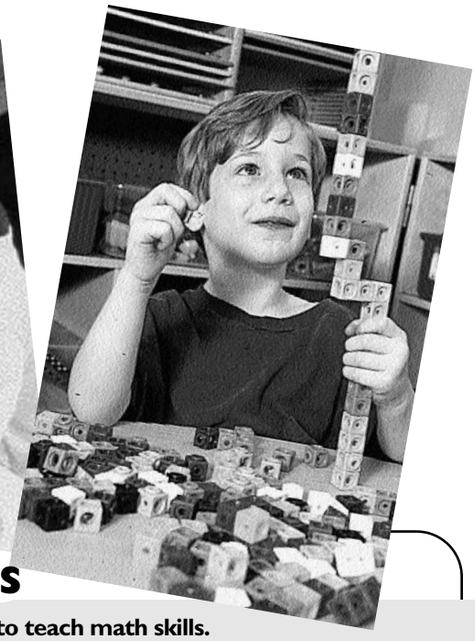

MATHEMATICS WITH **MANIPULATIVES**

Manipulatives such as those listed below can be used in many ways to teach math skills.

MATERIALS	DESCRIPTION	ACTIVITIES
Attribute Shapes	Shapes that come in logic sets, such as having all combinations of three colors, two sizes, two thicknesses, and six shapes	• Sorting • Matching
Cards	Similar to playing cards, these might have dots, numerals, or both	• Ordering the cards • Games such as war
Connecting cubes	Cubes in different colors that connect on one end	• Counting; building a set of stairs • Showing different ways to make a number (for instance, two red and two blue; three red and one blue)
Counters	Any small objects or two-sided beans	• Counting • Adding • Patterning
Pattern Blocks	A specific set of shapes that are easy to put together to make other shapes and patterns	• Making pictures • Extending and creating patterns • Shape puzzles • Making "number pictures"
Puppet	any puppet	• Play "Mr. Mixup" in which the puppet makes counting mistakes for the child to correct
Blocks	"Unit" or kindergarten blocks	• Counting; measuring; making symmetry buildings; patterns
Feely Box	A good feely box is made of tough cardboard, with two holes on opposite sides into which is sewn a tube sock with the toes cut off (making a "tunnel" open on both ends)	• Identifying shapes by touch • Counting objects only by touch
Tiles or other squares	The tiles/squares can be ceramic	• Making "number pictures" • Patterning

BOOK BOX

1 Hunter by Pat Hutchins (William Morrow, 1986; $16.95)

Anno's Counting Book by Mitsumasa Anno (HarperCollins, 1992; $22.99)

Bat Jamboree by Kathi Appelt (Scott Foresman, 1998; $5.95)

Color Zoo by Lois Ehlert (HarperCollins, 1992; $7.99)

Count and See by Tana Hoban (Simon & Schuster, 1972; $17)

Fish Eyes: A Book You Can Count On by Lois Ehlert (Harcourt, 2001; $6.95)

The Grouchy Ladybug by Eric Carle (Scott Foresman, 1996; $7.99)

How Many Bugs in a Box? by David A. Carter (Little Simon, 1988; $13.95)

How Many Snails? by Paul Giganti, Jr. (Scott Foresman, 1994; $5.95)

I Spy Two Eyes: Numbers in Art by Lucy Micklethwait (Mulberry, 1998; $10.99)

The Icky Bug Counting Book by Jerry Pallotta (Charlesbridge, 1992; $16.95)

Inch by Inch by Leo Lionni (Scott Foresman, 1995; $5.99)

My Very First Book of Shapes by Eric Carle (HarperCollins, 1985; $4.95)

On the Stairs by Julie Hofstrand Larios (Front Street, 1999; $15.95).

One Was Johnny by Maurice Sendak (HarperCollins, 1991; $5.99)

Over, Under and Through and Other Spatial Concepts by Tana Hoban (Simon & Schuster, 1973; $17)

The Right Number of Elephants by Jeff Sheppard (HarperCollins, 1992; $6.99)

The Shape of Things by Dayle Ann Dodds (Scott Foresman, 1996; $5.99)

Ten Black Dots by Donald Crews (William Morrow, 1986; $15.99)

Ten, Nine, Eight by Molly Bang (Tupelo, 1998; $6.99)

The Very Hungry Caterpillar* by Eric Carle (Scholastic Inc.; $5.95)

*To order; call 800-SCHOLASTIC.